We analyze the evolution of the polarization state of a signal in a recirculating loop with polarization-dependent loss. We show that the polarization-state evolution in experiments is in qualitative agreement with our analysis, and we discuss the relationship between the polarization-state evolution and the $Q$ factor. © 2002 Optical Society of America

Polarization-state evolution in recirculating loops with polarization-dependent loss

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Fiber recirculating loops have been extensively used as a relatively inexpensive way to study the accumulation of chromatic dispersion and nonlinearity and their interaction with other physical phenomena in long-haul transmission systems. Whereas loops have often been successful predictors of the behavior of straight-line systems with comparable parameters, it has been known for some time that, because of the periodicity of the system, the behavior of polarization phenomena in a loop can be quite different from that in a comparable straight line system. For example, it has been demonstrated that, when polarization controllers in a loop are properly set, polarization-dependent loss (PDL) can lead to the polarization of noise and to the elimination of noise that is orthogonal to the signal. To break the periodicity, loop-synchronous polarization scrambling has been used to emulate more closely the polarization behavior of straight-line systems. So far, most research reported in the literature has focused on the difference in performance between recirculating loops and straight-line systems. In this Letter we explain this difference and collect values of the corresponding eigenvalues $\lambda$. An eigenvector of $M_{\text{loop}}$ is also an eigenvector of $T$, and, if $\lambda$ is an eigenvalue of $M_{\text{loop}}$, then $\lambda^n$ is an eigenvalue of $T$. It follows that, if our input is an arbitrary field of the form

$$ u_{\text{in}} = c_+ u_+ + c_- u_- , \quad (1) $$

then the output field vector is

$$ u_{\text{out}} = Tu_{\text{in}} = c_+ \lambda^n u_+ + c_- \lambda^{-n} u_- . \quad (2) $$

If we let $\lambda_\pm = |\lambda_\pm|\exp(i\phi_\pm)$ and $c_\pm = |c_\pm|\exp(i\psi_\pm)$, then the corresponding output Stokes vector is

$$ s_{\text{out}} = (u_{\text{out}}^\dagger \sigma_3 u_{\text{out}}, u_{\text{out}}^\dagger \sigma_1 u_{\text{out}}, u_{\text{out}}^\dagger \sigma_2 u_{\text{out}})^T , $$

yielding

$$ s_{\text{out}} = |c_+|^2 |\lambda_+|^2 s_+ + |c_-|^2 |\lambda_-|^2 s_- + 2|c_+||c_-| (|\lambda_+||\lambda_-|)^2 \text{Re}(s_\text{out} \exp[-i(n\Delta \phi + \Delta \psi)]) , \quad (3) $$

where $s_\pm = (u_+^\dagger \sigma_3 u_+, u_-^\dagger \sigma_3 u_-, u_+^\dagger \sigma_1 u_+, u_-^\dagger \sigma_1 u_-, u_+^\dagger \sigma_2 u_+, u_-^\dagger \sigma_2 u_-)$, $s_\pm = (u_+^\dagger \sigma_3 u_+, u_-^\dagger \sigma_3 u_-, u_+^\dagger \sigma_1 u_+, u_-^\dagger \sigma_1 u_-, u_+^\dagger \sigma_2 u_+, u_-^\dagger \sigma_2 u_-)^T$, $\Delta \phi = \phi_+ - \phi_-$, and $\Delta \psi = \psi_+ - \psi_-$. In the above expressions, $\sigma_\pm$ represent the standard Pauli spin matrices. In Eq. (3), if the eigenvalues have different magnitudes, then

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one obtains spiral motion. The spiral centers are given by one of the polarization eigenstates, \( s_\pm \), and the rotation is described by the third term, where the rotation angle of the spiral that is due to one round trip of the recirculating loop is given by the phase difference of the eigenvalues, \( \Delta \phi \). The ratio of the magnitudes of the eigenvalues gives the relative decay rate of each of the coefficients, thus yielding the rate of convergence to a spiral center.

To understand the spiral behavior in a loop with PDL we consider a transfer matrix of the form

\[
M_{\text{loop}} = M_{\text{PDL}} M_{\text{rot}},
\]

where \( M_{\text{rot}} \) is a unitary matrix representing a fixed rotation on the Poincaré sphere and \( M_{\text{PDL}} \) represents the effect of PDL in one round trip of the loop. Inasmuch as an arbitrary rotation on the Poincaré sphere can be expressed as a rotation through an angle \( 2\gamma \) about an axis given by a unit vector \( \hat{s}_{\text{rot}} = (x, y, z)^T \), matrix \( M_{\text{rot}} \) can be expressed in the form

\[
M_{\text{rot}} = \begin{bmatrix}
\cos \gamma + i x \sin \gamma & (z - i y) \sin \gamma \\
(z + i y) \sin \gamma & \cos \gamma - i x \sin \gamma
\end{bmatrix}.
\]

The effect of PDL is modeled by the Jones matrix

\[
M_{\text{PDL}} = \begin{bmatrix}
1 & 0 \\
0 & 1 - \epsilon
\end{bmatrix},
\]

where the strength of the PDL is given by a small non-negative parameter \( \epsilon \) that is less than 1. Note that, in this notation, the low-loss axis of the PDL is given by \( \hat{s}_{\text{PDL}} = (1, 0, 0)^T \) in Stokes space.

We exploit the fact that \( \epsilon \) is small by using standard perturbation methods to expand our eigenvector in powers of \( \epsilon \), solving for corrections at each order.\(^5\) We begin by seeking eigenvalues and eigenvectors that satisfy \( (M_{\text{loop}} - \lambda I) u = 0 \), where \( I \) is the \( 2 \times 2 \) identity matrix, and expand \( \lambda \) and \( u \) in power series in \( \epsilon \) as \( \lambda = \lambda_0 + \epsilon \lambda_1 + O(\epsilon^2) \) and \( u = u_0 + \epsilon u_1 + O(\epsilon^2) \). From these expressions we can compute the Stokes eigenvector in powers of \( \epsilon \) as well, obtaining \( s_\pm = s_\pm^{(0)} + \epsilon s_\pm^{(1)} + O(\epsilon^2) \). The \( O(1) \) problem \( (M_{\text{rot}} - \lambda_0 I) u_0 = 0 \), which we obtain by setting \( \epsilon = 0 \), corresponds to a rotation when the effect of PDL is neglected. The leading-order eigenvalues are \( \lambda_0 = \exp(\pm i \gamma) \), and the leading-order eigenvectors are

\[
u_0 = \left( \frac{z - i y}{i(\mp 1)} \right),
\]

where we have normalized these eigenvectors such that they have unit length. It is straightforward then to show that these eigenvectors in Jones space yield the corresponding Stokes vectors \( s_\pm^{(0)} = \pm \hat{s}_{\text{rot}} \), as expected in the \( O(1) \) problem.

The \( O(\epsilon) \) eigenvalue correction is given by \( \lambda_1 = -\frac{1}{2}(1 \mp x)\exp(\pm i \gamma) \). Similarly, the \( O(\epsilon) \) correction to the eigenvector satisfies the generalized eigenvector equation

\[
(M_{\text{rot}} - \lambda_0 I) u_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} M_{\text{rot}} + \lambda_1 I u_0,
\]

which yields

\[
u_1 = i k u_0 + u_1' = \frac{i k u_0 + \csc \gamma}{2\sqrt{2}} \exp(\pm i \gamma) \sqrt{1 + x} \left( \begin{array}{c} 1 \mp x \\ z \cot \gamma \mp y \\ -y \cot \gamma \mp z \end{array} \right),
\]

where \( k \) is a real constant whose value is not determined until the \( O(\epsilon^2) \) level. The \( O(\epsilon) \) correction to the eigenvector in Stokes space is then given by

\[
\hat{s}_{\text{rot}} = \pm \hat{s}_{\text{rot}}^{(0)} \mp \frac{1}{2} \exp(\pm i \gamma) \hat{s}_{\text{PDL}} \cot \gamma + O(\epsilon^2).
\]

Note that, because of the \( O(\epsilon) \) corrections, the eigenstates in Stokes space are not antiparallel in the presence of PDL, unlike the principal states for PMD.\(^6\)

To obtain the explicit form of the output Stokes vector in the loop model we still need expressions for \( \lambda_{\pm} \) and \( s_\pm \). We use the asymptotic expansion; the coefficients in Eq. (3) are given by \( A_{\pm} = |\lambda_{\pm}|^2 = 1 - \epsilon(1 \mp x) + O(\epsilon^2) \) and \( B = |\lambda_{\pm}| |\lambda_{\mp}| = 1 - \epsilon + O(\epsilon^2) \). As \( A_{\pm} \) and \( B \) are less than 1, \( A_{\pm}^{-1} \) and \( B^{\frac{1}{2}} \) decay as \( n \to \infty \). The slowest-decaying term of the three terms in Eq. (3) is the dominant term, which determines the attracting eigenstate. The relative decay rate of each of the terms in Eq. (3) is determined by the inequalities \( A_{\pm} \approx B \approx A_{\pm} \), which hold when \( x \approx 0 \). Thus, if the low-loss axis of the PDL \( \hat{s}_{\text{PDL}} \) is in the same hemisphere as rotation axis \( \hat{s}_{\text{rot}} \), the \( s_+ \) eigenstate is the attracting eigenstate. The converse is also true. We conclude that, if \( x \neq 0 \), we have spiral motion toward one of the eigenstates but, if \( x = 0 \), all terms remain comparable and we observe nearly circular behavior on the Poincaré sphere. From the expressions for \( \lambda_{\pm} \) and \( \lambda_1 \) it follows that

\[
\exp(\pm i \phi_{\pm}) = \lambda_{\pm}/|\lambda_{\pm}| = \exp(\pm i \gamma) + O(\epsilon^2),
\]

so that, to \( O(\epsilon^2) \), \( \Delta \phi = 2\gamma \). We also obtain

\[
s_\pm = (i \hat{t}_1 - \hat{t}_2)(1 - \epsilon/2)(1 + i \cot \gamma) + O(\epsilon^2),
\]

where \( \hat{t}_1 = (\hat{s}_{\text{rot}} \times \hat{s}_{\text{PDL}})/|\hat{s}_{\text{rot}} \times \hat{s}_{\text{PDL}}| \) and \( \hat{t}_2 = (\hat{s}_{\text{rot}} \times \hat{s}_{\text{PDL}})/|\hat{s}_{\text{rot}} \times \hat{s}_{\text{PDL}}| \). Therefore the output Stokes vector is
$s_{\text{out}} = A_+^n |c_+|^2 s_+ + A_-^n |c_-|^2 s_-$

\[ + 2B^n |c_+| |c_-| (\hat{t}_1 \sin \xi_n - \hat{t}_2 \cos \xi_n) \]

\[ + (\epsilon/2) (\hat{t}_1 (\cot \gamma \cos \xi_n - \sin \xi_n)) \]

\[ + \hat{t}_2 (\cos \xi_n + \cot \gamma \sin \xi_n)) + O(\epsilon^2), \quad (14) \]

where $\xi_n = 2n\gamma + \Delta \psi$.

Neglecting the $\epsilon$ contributions in Eq. (14), we note that the $B^n$ term contains sinusoidal pieces that correspond to the spiral or circular motion on the Poincaré sphere. We compare the spiral behavior from the experimental results shown in Fig. 1 with the analytical result in Eq. (14). In Eq. (14) we have an inward spiral evolution if $A_+ > B > A_-$ and the input polarization state is closer to the attracting $s_+$ eigenstates, i.e., $|c_+| > |c_-|$, so the signal’s polarization state remains on the same hemisphere as the low-loss axis of the PDL throughout transmission, corresponding to high $Q$. This situation is illustrated in Fig. 1(a), in which the center of the spiral is the attracting eigenstate. Conversely, we have an outward spiral if $A_- > B > A_+$ and the input polarization state is close to the repelling $s_-$ eigenstate, i.e., $|c_-| < |c_+|$. In this case the signal’s polarization state can remain for a long time on the same hemisphere as the high-loss axis of the PDL, particularly if $|c_-| \ll |c_+|$, corresponding to a low $Q$. This situation corresponds to Fig. 1(b), in which the center of the spiral is the repelling eigenstate. Finally, if $A_+ \approx B \approx A_-$, then there is a slow convergence because no single term dominates. In this case the circle’s center is orthogonal to the low-loss axis of the PDL, and the signal passes alternately through the high-loss and low-loss axes of the PDL, corresponding to a medium $Q$. This situation corresponds to the polarization evolution shown in Fig. 1(c), in which there seems to be no attracting or repelling state.

The experimental and theoretical results show that PDL plays a major role in the loop performance and polarization-state evolution. In recirculating loop systems, the PDL will unrealistically improve the performance when the polarization evolution is optimized by use of a polarization controller. The noise tends to polarize in the polarization state of the signal, and the noise that is orthogonal to the signal is reduced. We have characterized the behavior of the polarization state as a spiral evolution on the Poincaré sphere and have associated different types of spiral with different system performance.

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