Wavelength Division Multiplexing in an Unfiltered Soliton Communication System

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Abstract—The limits on the minimum and maximum channel spacing of an unfiltered wavelength division multiplexed soliton communication system are examined. It is shown that soliton frequency shifts due to collisions at the receiver limit the minimum channel spacing to about four soliton spectral widths while the gain deviation among different channels sets a limit on the maximum channel spacing. The gain mismatch tolerance is determined.

I. INTRODUCTION

The invention of the erbium fiber amplifier [1], [2] made wide-band, all-optical long-distance fiber transmission line feasible. We can utilize the large gain bandwidth of the erbium amplifiers by multiplexing several signals, each at a different carrier wavelength, onto a single fiber link using wavelength division multiplexing (WDM). Solitons, with their demonstrated potential for WDM [3], are an attractive choice as the transmission medium in an all-optical, long-distance transmission line. It is therefore important to determine the fundamental limits on the minimum and maximum channel spacing in a soliton WDM system. To date, it is commonly believed that the timing jitter due to collisions of solitons at different channels [4], [5] determines the minimum channel spacing while the amplification period determines the maximum channel spacing [3]. In this paper, however, we shall show that soliton frequency shifts and gain imbalance among different channels can impose more stringent limits on the minimum and maximum channel spacing in an unfiltered soliton system. While there is great interest at the present time in filtered systems because it has been demonstrated that sliding filters greatly reduce the soliton timing jitter, there are reasons for investigating the unfiltered systems. First, the unfiltered system provides a baseline against which to examine the advantages and the disadvantages of the filtered system. Second, the filtered system is somewhat more complex than the unfiltered system to implement, so that the unfiltered system may be of use over distances of several thousand kilometers.

When two solitons at different carrier wavelengths approach each other, their relative velocities increase due to cross-phase-modulation so that their frequency spectra are pushed apart [6]. Consequently, the spectral energy of one channel can be pushed into the adjacent channel. When the solitons move apart, their relative velocities decrease to the original values and their spectra return to their original shapes. However, if the transmission line ends and the solitons enter a receiver while in the middle of a collision, the spectra of the solitons do not have a chance to return to normal and the frequency shift can change the amount of energy detected in a channel and enhance the probability of a detector error. Since the collision-induced frequency shift increases when the channel separation decreases, this frequency shift will impose a limit on how narrow the channel spacing can be.

Because the gain profile of erbium amplifier varies with wavelength while fiber loss only has a weak dependence on wavelength in the spectral range of interest, we can only set the gain of two channels on opposite sides of the maximum to match the loss. Other channels will experience either overall gain or loss. Consequently, soliton parameters for these gain-mismatched channels such as the amplitudes and widths will change as the solitons propagate along the fiber. The maximum tolerable deviation in the width of the soliton and the local gain profile will then determine the limit on how wide the channel spacing can be.

Both of the physical phenomena that we are discussing can be avoided at least in theory. By carefully choosing the length of the transmission line, the frequencies of the soliton channels, and the original temporal locations of the solitons in each channel, it is possible to arrange things so that solitons never collide at the receiver, eliminating this limit on the minimum channel spacing. However, that is cumbersome to implement and control. By flattening the gain curve of the Er-doped fiber amplifiers using grating filters or other techniques, it is possible to make the limit on the maximum channel spacing due to gain deviations less stringent than the limit due to soliton collisions. Gain flattening is important for both nonreturn-to-zero and soliton communication systems and hence is the subject of considerable investigation. However, it remains expensive and difficult to implement. Thus exploring both of these physical phenomena remains important.

The remainder of this paper is organized as follows. In Section II, we determine the minimum channel spacing set by soliton collisions in the receiver. In Section III, we calculate the effect of gain imbalance and determine the limit that it imposes on the maximum channel spacing. Section IV contains the conclusions.

II. MINIMUM CHANNEL SPACING

In this section, we determine the minimum channel spacing due to soliton collisions at the receiver in an unfiltered
transmission line. We measure the effect of the collisions by calculating the minimum contrast ratio, defined as the ratio of the minimum energy measured in a 1 to the maximum energy measured in a 0. In this calculation, we do not take into account the gain deviation, so that the gain balances the loss in all channels. The equation governing the slowly varying electric field envelope \( q(z,t) \) is given by

\[
\frac{\partial q}{\partial z} + \frac{1}{2} \frac{\partial^2 q}{\partial t^2} + |q|^2 q = 0
\]  

(1)

where \( z \) is normalized distance along the fiber and \( t \) is normalized time. We assume that the channels are equally spaced and that solitons in different channels have the same amplitudes so that the input to the fiber is

\[
q(0,t) = \sum_{n=1}^{N} \delta_n \text{sech} \theta_n \exp(iV_n t_n - iV_n t_n/2 + i\phi_n)
\]  

(2)

where \( \theta_n = t - V_n z + t_n \), \( N \) is the number of channels, \( t_n \) is the initial temporal location of the \( n \)th soliton, \( V_n = [(N+1)/2 + n] \Omega \) is the central frequency of the \( n \)th channel given that \( \Omega \) is the channel spacing, \( \phi_n \) is the intrinsic phase of the \( n \)th soliton, and \( \delta_n = 1 \) if the \( n \)th channel is filled while \( \delta_n = 0 \) if the \( n \)th channel is empty, assuming that the solitons all have amplitude 1. The initial temporal locations \( t_n \) of the solitons are chosen so that the solitons are initially far apart. If the solitons overlap initially, the partial collision will result in permanent frequency shifts for the solitons and significant increases in their timing jitter. We choose the ratio \( t_n/V_n \) to be constant which implies that all \( N \) channels collide simultaneously, and we add the phase factor \( \exp(-iV_n t_n/2) \) so that the solitons will have the same phase if their intrinsic phases are the same and there are no collisions.

The energy measured in the \( n \)th channel \( E_n(z) \) is defined as

\[
E_n(z) = \int_{-\infty}^{\infty} d\omega |g_n(\omega)| \langle \hat{q}(z,\omega) \rangle^2
\]  

(3)

where \( \omega \) is the frequency, \( \hat{q}(z,\omega) \) is the spectrum of the electric field envelope \( q(z,t) \) at a distance \( z \), and \( g(\omega) \) is the spectral function of the receiver. In our simulations, we have used both an ideal detector

\[
g_n(\omega) = \begin{cases} 
1, & |\omega - V_n| < W \\
0, & \text{otherwise}
\end{cases}
\]  

(4)

and a detector with a Gaussian spectrum

\[
g(\omega) = \exp[-(\omega - V_n)^2/2W^2]
\]  

(5)

where \( 2W \) is the width of the measurement window of the detector. In the simulations to be presented in this paper, we used the ideal detector (4) but we found that the results are nearly the same for both detectors. The energy \( E_n(z) \), and hence the minimum contrast ratio, will depend on the width of the measurement window \( 2W \), the intrinsic phases of the solitons \( \phi_n \), and the width of the channels \( \Omega \).

To determine the minimum channel width in an \( N \)-channel soliton WDM system, we first determine the soliton pattern arriving at the receiver that will give the minimum contrast ratio. We have found that the minimum energy in a 1 will be detected when all the channels are filled with colliding solitons at the receiver. The highest wavelength and lowest wavelength channels will have the minimum energy during the collision since all the other channels tend to shift the frequency of the solitons in these two channels in the same direction. Similarly, the maximum energy in a 0 will be detected when all but one channel is filled with colliding solitons at the detector and the empty channel is at either the highest or lowest wavelength. We have carried out numerical simulations with up to eight channels to verify these observations.

We now describe the dependence of the minimum contrast ratio on the spectral width of the detector window using a 3-channel system as a specific example. Fig. 1 plots the minimum contrast ratio for a 3-channel WDM system versus the spectral width of the detector window. The solid line corresponds to the contrast ratio for \( \Omega = 2\Delta \omega \) and the dashed line corresponds to the contrast ratio for \( \Omega = 2.5\Delta \omega \). The maximum energy of the 0 is obtained from the pattern 110 while the minimum energy of the 1 is obtained from the pattern 111. The temporal separation between solitons in adjacent channels is 10\( \tau \) and \( \phi_n = 0 \).

![Fig. 1](image-url)  

**Fig. 1.** The minimum contrast ratio for a 3-channel WDM system versus the spectral width of the detector window. The solid line corresponds to the contrast ratio for \( \Omega = 2\Delta \omega \) and the dashed line corresponds to the contrast ratio for \( \Omega = 2.5\Delta \omega \). The maximum energy of the 0 is obtained from the pattern 110 while the minimum energy of the 1 is obtained from the pattern 111. The temporal separation between solitons in adjacent channels is 10\( \tau \) and \( \phi_n = 0 \).
the slowest soliton is injected into the fiber first and the fastest soliton is injected last, one finds after all the collisions have taken place and the solitons are far apart that the total phase change of the nth soliton is given by [7]

\[
\Delta \phi_n = \phi^+_n - \phi^-_n = 2 \sum_{k=1}^{n-1} \text{Arg} \left( \frac{\zeta_n - \zeta_k}{\zeta^n_n - \zeta^k_k} \right) - 2 \sum_{k=n+1}^{N} \text{Arg} \left( \frac{\zeta_n - \zeta_k}{\zeta^n_n - \zeta^k_k} \right)
\]

(6)

where \( \phi^+_n \) and \( \phi^-_n \) are the intrinsic phases of the nth soliton before and after the collision, \( \zeta_n = (V_n + i)/2 \), where we recall that the solitons have amplitude 1 and velocity \( V_n \), and \( \text{Arg}(x) \) is the argument of the complex number \( x \). It follows from (6) that two solitons will not in general be in phase during a collision even if they are initially in phase.

In an \( N \)-channel WDM system with equal channel spacing and equal soliton amplitudes, the intrinsic phase changes of the solitons in the \( n \)th and \((N - n)\)th channels will be the same. Since only the phase differences between the channels are important, we can set the intrinsic phase of the center channel to be zero if \( N \) is odd or the intrinsic phases of the middle two channels to be zero if \( N \) is even. We therefore have \((N - 1)/2\) independent phase parameters when \( N \) is odd or \((N/2) - 1\) independent phase parameters when \( N \) is even. Since the energy measured in a channel depends on both the collision-induced frequency shift and the interference of the soliton spectra, the intrinsic phase arrangement of the solitons that will give the maximum energy in a 0 or the minimum energy in a 1 has to be determined numerically. In Fig. 2, we plot the minimum energy in a 1 (solid line) and the maximum energy in a 0 (dashed line) for a three-channel WDM system versus the intrinsic phase \( \phi_1 \). The energy for the 1 is calculated from the pattern 111 while the energy for the 0 is calculated from the pattern 110. The channel width is \( 2\Delta \omega \) and the detector spectral width is 0.25\( \Delta \omega \). From Fig. 2, the energy for the 1 reaches its minimum at \( \phi_1 \approx 0.26\pi \) and \( \phi_1 \approx 1.7\pi \), which is about 20% smaller than the energy of the 1 at \( \phi_1 = 0 \). From Fig. 2, the maximum energy for the 0 is obtained when the solitons are all initially in phase.

Fig. 3 shows the minimum contrast ratio versus the channel width for a 3-channel WDM system. The channel width is normalized to the soliton spectral width. As the channel width decreases, the energy that extends into an empty channel from a soliton in an adjacent channel decreases exponentially and the collision-induced frequency shift also decreases so that less energy is pushed into the empty channel. The change in the energy in a 1 is comparatively small. Therefore the contrast ratio increases when the channel width increases. In Fig. 4, we plot the minimum contrast ratio versus the number of channels. The solid circles correspond to \( \Omega = 2\Delta \omega \), the solid squares correspond to \( \Omega = 3\Delta \omega \), and the crosses correspond to \( \Omega = 4\Delta \omega \). The largest number of channels that we studied was eight which implies that at most three independent phase parameters had to be varied. When the number of channels increases, the minimum contrast ratios decrease because the increase in the number of collisions results in a larger frequency shift of the solitons. The additional frequency shift of the solitons due to the soliton in the added channel however is smaller because of the increase in frequency separation. From Fig. 4, the contrast ratio for \( \Omega = 4\Delta \omega \) levels off at 50. Further increase in the number of channels has no effect on the contrast ratio. If we assume a system requirement for the minimum contrast ratio to be 20, the minimum channel width for a soliton WDM system is about 4 soliton spectral widths.

The minimum channel spacing measured with respect to the soliton spectral width imposed by the collision-induced frequency shift at the receiver is independent of the width of the soliton. By comparison, the minimum channel spacing imposed by collision-induced temporal offsets depends on the maximum allowable timing offset. Consider a recent 8 x 2.5 Gb/s soliton WDM transmission experiment over 10,000 km with \( \tau = 60 \) psec [8]. The average dispersion coefficient \( D \) is 0.6 ps/nm/km and the adjacent channel spacing is 0.2 nm or 4.8\( \Delta \omega \). In this system, the solitons pass through three time slots in a neighboring frequency channel. When one soliton follows another, the second soliton will pass through two of the same time slots as the first, but the third will be different. Consequently, the temporal offsets that are experienced by a
train of solitons are partially correlated, but the uncorrelated portion will lead to timing jitter. Reasonable estimates of the maximum allowable timing jitter are in the range of 50–100 ps [9] which implies an allowable channel spacing of $2\Delta \omega - 4\Delta \omega$ [10]. This limitation is comparable to the limit due to collisions at the receiver but is somewhat smaller.

III. MAXIMUM CHANNEL WIDTH

Next we determine both analytically and numerically the limits on the maximum channel separation in a soliton WDM system imposed by gain imbalance among different channels. Because the gain profile of the erbium amplifier varies with wavelength while fiber loss only has a weak dependence on wavelength in the spectral range of interest, it is only possible for two channels in a WDM system on opposite sides of the gain maximum to have the fiber loss exactly balanced by the gain. All other channels will have either excess gain or insufficient gain, thus resulting in gain imbalance. When there is excess gain in a channel, the energy of the soliton increases every time it passes through an amplifier. The temporal duration of the soliton decreases and hence its spectral width increases. The spectrum of the soliton will therefore extend into adjacent channels and affect the observed contrast ratio. When a channel has insufficient gain, the energy of the soliton decreases as it travels down the fiber and its soliton period increases. Eventually the attenuation length due to the gain deviation becomes comparable to the soliton period, and the soliton can no longer adjust for the excess loss. Depending on the local gain profile of the erbium amplifier, the maximum tolerable gain deviation will set a limit on the maximum channel spacing of a WDM system.

To include the effect of fiber loss and lumped amplifiers, (1) becomes

$$i\frac{\partial q}{\partial z} + \frac{1}{2} \frac{\partial^2 q}{\partial t^2} + |q|^2 q = -i\Gamma q + iG \sum_{n=1}^{N_a} \delta(z - n L_a) q$$

(7)

where $2\Gamma$ is the loss coefficient and $G$ is the discrete gain, $N_a$ is the number of amplifiers in a fiber length $L_a$, $L_a$ is an amplification period, $N_a L_a \leq Z < (N_a + 1) L_a$, and $\delta(z)$ is the Dirac delta-function. The energy of a soliton at a distance $z$ is given by

$$I(z) = I_0 \exp \left[ 2 \int_0^z dz' f(z') \right]$$

(8)

where $f(z) = -\Gamma + G \sum_{n=1}^{N_a} \delta(z - n L_a)$ and $I(z) = \int_{-\infty}^{z} |q|^2 dt$ is the energy of the pulse. If we demand that the energy of the pulse be restored at the end of each amplifier, we obtain $G = \Gamma L_a$. The distance in (7) is normalized to the dispersion length scale $L_d$ which is typically hundreds of kilometers. Since the amplification period $L_a$ is on the order of 50 km, the ratio $\epsilon = L_a / L_d$ is typically small. With this normalization, the loss coefficient $\Gamma$ is typically large, of the order $\epsilon^{-1}$. The short amplifier spacing means that the soliton is amplified many times in a distance $L_d$. In other words, the coefficient $f(z)$ is large but varies rapidly. Using a multiple length scale analysis [11], we obtain

$$i\frac{\partial q^{(0)}}{\partial z} + \frac{1}{2} \frac{\partial^2 q^{(0)}}{\partial t^2} + \rho |q^{(0)}|^2 q^{(0)} = i\Delta G q^{(0)}$$

(9)

where $\rho = [1 - \exp(-2\Gamma L_a)]/2\Gamma L_a$, $\Delta G$ is the gain deviation

$$q(z,t) = \sum_{n=0}^{\infty} \epsilon^n q^{(n)}(z,t),$$

$$q^{(0)}(z,t) = \tilde{q}^{(0)}(z,t) \exp[-\Gamma(z - n L_a)]$$

(10)

and $n L_a \leq z < (n + 1) L_a$.

If there is no gain deviation, so that $\Delta G = 0$, the fundamental soliton solution of (9) is

$$\tilde{q}^{(0)}(z,t) = \frac{1}{\sqrt{\rho}} A \mathrm{sech}(A(t - \Omega z))$$

$$\times \exp \left\{ i \omega t + \frac{1}{2} (A^2 - \Omega^2) z \right\}$$

(11)

where $A$ is the amplitude and $\Omega$ is the central frequency of the soliton. With a small gain variation, $|\Delta G| \ll G$, the amplitude $A$ becomes distance-dependent so that

$$A(z) = A_0 \exp(2\Delta G z/L_a)$$

(12)

where $A_0$ is the initial soliton amplitude. The effect of a small gain deviation $\Delta G$ is therefore an exponential change of the soliton amplitude and hence its spectral width. The first order correction is given by

$$\tilde{q}^{(1)}(z,t) = \left( \frac{1}{2i\Gamma} \left[ 1 - \exp \left\{ -2\Gamma(z - n L_a) \right\} \right] - \rho(z - n L_a) \right)$$

$$\times \tilde{q}^{(0)}(z,t) \exp(i \Delta G z/L_a) \tilde{q}^{(0)}(z,t)$$

(13)

We simulated the propagation of 50 ps pulses with a period $z_0 \sim 1000$ km over a total distance of 9000 km with amplifiers spaced 50 km apart. The fiber loss is 0.2 dB/km. The gain mismatch is modeled by having an additional gain factor $R = \exp(2\Delta G z/L_a)$ at each amplifier. We studied the evolution of the solitons for different values of $R$ and evaluated the changes in the soliton amplitude. In Fig. 5, we plot the soliton amplitudes at the end of 9000 km versus $R$. The final amplitude of the soliton no longer evolves exponentially.
when $R \geq 0.035$ dB. When there is excess gain, the soliton amplitude increases and the soliton period which equals $L_d$ decreases quadratically. When the soliton grows to 3 times its initial amplitude, the soliton period is approximately equal to the amplification period $L_a$ and the multiple length scale expansion breaks down. The frequency sidebands generated due to the excess gain [12] grow and multiple solitons form. When gain cannot balance loss, the soliton amplitude, averaged over one amplification period, decreases. When the attenuation length of the averaged soliton becomes comparable to the soliton period, the soliton cannot reshape itself to adjust for the attenuation. The soliton amplitude begins to decrease according to $A(z) = A_0 \exp(-\Delta z / L_a)$ rather than (12) and hence is larger than the predicted values.

For $R = 0.016$ dB per amplifier, the soliton amplitude after propagation of 9000 km increases by a factor of 2. Hence, a small variation in the gain of an amplifier can cause large changes in the transmitted pulse width. Using the local gain profile of the erbium amplifier, the gain deviation sets a limit on the maximum channel spacing. As an illustration, we assume a 3 dB gain bandwidth of 20 nm [13] around the broad peak at 1558 nm for a gain of about 10 dB. If we assume that the maximum tolerable change in the soliton amplitude is about a factor of two excess gain or loss for a total factor of four, an assumption that is consistent with our numerical results, and we use a quadratic approximation for the gain spectrum, the maximum channel separation is 1.5 nm. By comparison, the maximum channel separation imposed by the amplification period is 2 nm. The maximum channel spacing in a recent 8 × 2.5 Gb/s soliton WDM experiment [8] is 1.4 nm which agrees with our calculations, but that is probably fortuitous as the experiments use some amount of gain flattening [9].

IV. CONCLUSION

We have found the minimum channel width in an unfiltered soliton WDM system due to interchannel soliton collisions at the receivers. For a minimum contrast ratio of 20, we find that the minimum channel width is about four soliton FWHM spectral widths, which is 0.17 nm for a 60 ps pulse. We also found that the maximum tolerable excess gain or loss for a soliton is about 0.016 dB in a system with a chain of 180 amplifiers if the soliton duration is not to change by a factor of two. These results are consistent with the experimental data of Nyman et al. and provide a baseline for future investigations on the advantages and disadvantages of filtered systems.

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[10] In [3], the maximum number of collisions is underestimated by a factor of 2. Equation (5) in [3] should read $\Delta t = \pm 2.638(1/\Delta f_s)$. In calculating the minimum channel spacing, the maximum time shift, not the average time shift, should be used.

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