Optical Modulation Formats in PhoSSiL

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PhoSSiL supports a variety of optical modulation formats. These are enumerated and documented in enum typePulse in the file ModulationFormats.hh. In this document we reproduce that material in a way that is hopefully easier to assimilate.

The formats are in three groups:

(I) Formats that are given by simple analytical mathematical formulae but that are not necessarily experimentally realistic. Typically these provide acceptable pulse shapes in the time domain. However, they tend not to be very accurate in the frequency domain. These formats (numbered with the values used to set them in Signal.in) are

(1) NRZ
(2) RZ_GAUSS
(3) RZ_SECH
(4) CRZ_GAUSS
(5) CRZ_RCOS
(6) CRZ_SIN_MZ
(11) QPSK_CW
(12) QPSK_RZ_GAUSS

(II) Formats that are constructed using a phase or Mach-Zehnder modulator. These formats are experimentally realistic but do not always have simple mathematical formulae. They are much more accurate representations of experimentally generated formats than those in Group I, especially in the frequency domain. These formats are

(14) NRZ_RCOS_MZ
(15) RZ33_MZ
(16) RZ50_MZ
(17) RZ67_MZ
(18) BPSK_PM_NRZ
(19) BPSK_PM_RZ33
(20) BPSK_PM_RZ50
(21) BPSK_PM_RZ67
(22) BPSK_MZ_NRZ
(23) BPSK_MZ_RZ33
(24) BPSK_MZ_RZ50
(25) BPSK_MZ_RZ67
(26) QPSK_SEQ_PM_NRZ
(27) QPSK_SEQ_PM_RZ33
(28) QPSK_SEQ_PM_RZ50
(29) QPSK_SEQ_PM_RZ67
(30) QPSK_DPMZ_NRZ
(31) QPSK_DPMZ_RZ33
(32) QPSK_DPMZ_RZ50
(33) QPSK_DPMZ_RZ67
(34) ODB

(III) Miscellaneous special-purpose formats

(8) CW
(13) INPUT_SIGNAL_FROM_FILE
(9) RZ_GAUSS_PATTERN
(10) INPUT_SEQUENCE
(7) UMBC_RZ

**Group II Formats**

The electrical signal, $V = V(t)$, is constructed by smoothing a binary signal. The types of smoothing, which are specified using TypeEOMElecData, are

(1) EOM_BINARY_DATA_PERFECT_RECT
(2) EOM_BINARY_DATA_ARCTANH_SMOOTHING (Parameter: EOM_RiseTime)
(3) EOM_BINARY_DATA_SUPERGAUSS_SMOOTHING (Parameter: EOM_SuperGaussExponent)
(4) EOM_BINARY_DATA_RCOS (Parameter: Either optical RiseTime or electrical EOM_RiseTime. Set the one you don’t want to be negative.)

The resulting electrical signal can then be filtered using an electrical filter with TypeEOMElecFilter being one of

(0) EOM_NO_ELEC_FILTER

(1) EOM_GAUSSIAN (Parameter: EOM_ElecFilter_FreqFWHM)

(2) EOM_BESSEL (4th-order, with parameter: EOM_ElecFilter_FreqFWHM)

The following two combinations are analyzed in Ho and Kahn, JLT, 22 (2), pp. 658-663, 2004.

• TypeEOMElecData = EOM_BINARY_DATA_RCOS
  with TypeEOMElecFilter = EOM_NO_ELEC_FILTER

• TypeEOMElecData = EOM_BINARY_DATA_PERFECT_RECT
  with TypeEOMElecFilter = EOM_BESSEL.

In the case TypeEOMElecData = EOM_BINARY_DATA_RCOS, we define

\[ V(t) = \sum_k b_k p(t - kT - T/2), \]  

where \( b_k \) is the binary data (either 0, 1 or \( \pm 1 \)) and \( T = \text{BitLength} \). The pulse shape \( p \) is defined in terms of a roll-off parameter, \( \beta \), that is computed from the rise time input parameter. When \( \beta > 0 \) the formula for \( p \) is

\[ p(t) = \begin{cases} 
1, & |t| \leq \frac{1-\beta}{2}T \\
0, & |t| \geq \frac{1+\beta}{2}T \\
\frac{1}{2} \left[ 1 - \sin \left( \frac{\pi |t|}{\beta T} \right) \right], & \text{otherwise},
\end{cases} \]  

and when \( \beta = 0 \),

\[ p(t) = \begin{cases} 
1, & |t| \leq \frac{T}{2}, \\
0, & |t| > \frac{T}{2}.
\end{cases} \]  

The optical signals, \( u = u(t) \), for the modulation formats in Group II are constructed from \( V \) as follows.
NRZ_RCOS_MZ

(See Ho and Kahn op. cit.) Parameters are

- TypeEOMElecData = EOM_BINARY_DATA_RCOS (set automatically in code)
- Either optical RiseTime or electrical EOM_RiseTime
- TypeEOMElecFilter = EOM_NO_ELEC_FILTER (set automatically in code)
- ExtinctionRatio in dB
- ChirpParameter.

Up to a constant factor, $u$ is given by

$$u_{\text{NRZ}}(t) = \frac{1}{2} \left[ \exp\left(\frac{i\pi}{4}(1 + \alpha)V(t)\right) + i\chi \exp\left(-\frac{i\pi}{4}(1 - \alpha)V(t)\right) \right], \quad (4)$$

where $\alpha = \text{ChirpParameter}$ and $\chi$ is computed from ExtinctionRatio.

RZ33_MZ

(See Ip and Kahn, JLT vol 24, no.3, March 2006, pp. 1610-1618.) Constructed by multiplying NRZ_RCOS_MZ by a train of 33% duty cycle optical pulses. Parameters are as for NRZ_RCOS_MZ. Formula for optical signal is

$$u(t) = u_{\text{NRZ}}(t) \sin\left[\frac{\pi}{2} (1 + \cos(\pi t/T))\right]. \quad (5)$$

RZ50_MZ

(See Ip and Kahn, JLT vol 24, no.3, March 2006, pp. 1610-1618.) Constructed by multiplying NRZ_RCOS_MZ by a train of 50% duty cycle optical pulses. Parameters are as for NRZ_RCOS_MZ. Formula for optical signal is

$$u(t) = u_{\text{NRZ}}(t) \sin\left[\frac{\pi}{4} (1 - \cos(2\pi t/T))\right]. \quad (6)$$

RZ67_MZ

(See Ip and Kahn, JLT vol 24, no.3, March 2006, pp. 1610-1618.) Constructed by multiplying NRZ_RCOS_MZ by a train of 67% duty cycle optical pulses. Parameters are as for NRZ_RCOS_MZ. Formula for optical signal is

$$u(t) = u_{\text{NRZ}}(t) \sin\left[\frac{\pi}{2} \sin(\pi t/T)\right]. \quad (7)$$
**BPSK_PM_NRZ**

Binary phase-shift keyed with constant power. Parameters are

- **TypeEOMElecData** with appropriate parameters (Recommendation: EOM_BINARY_DATA_RCS)
- **TypeEOMElecFilter** with appropriate parameters (Recommendation: EOM_NO_ELECFILTER)

Up to a constant factor, $u$ is given by

$$ u(t) = \exp[i\pi(1-V(t))]. \quad (8) $$

So

$$ u = \begin{cases} 
-1 & \text{if } V = 0, \\
1 & \text{if } V = 1. 
\end{cases} \quad (9) $$

Transitions between data symbols lie on the unit circle.

**BPSK_MZ_NRZ**

Binary phase-shift keying using a Mach Zehnder modulator (MZM). Parameters are

- **TypeEOMElecData** with appropriate parameters (Recommendation: EOM_BINARY_DATA_RCS)
- **TypeEOMElecFilter** with appropriate parameters (Recommendation: EOM_NO_ELECFILTER)
- $\epsilon = \text{MZH_MZMSubOptimalModulationDepthFactor}$ in range $0 \leq \epsilon < 0.25$. ($\epsilon = 0$ is optimal case.)

Up to a constant factor, $u$ is given by

$$ u(t) = \sqrt{P_0}\cos\left\{\pi [1 - \epsilon + (2\epsilon - 1)V(t)]\right\}. \quad (10) $$

So,

$$ u = \begin{cases} 
\sqrt{P_0}\cos(\pi - \epsilon)) \gtrsim -\sqrt{P_0} & \text{if } V = 0 \\
\sqrt{P_0}\cos(\epsilon) \lesssim \sqrt{P_0} & \text{if } V = 1. 
\end{cases} \quad (11) $$

Transitions between data symbols go through the origin.

**QPSK_SEQ_PM_NRZ**

Sequential binary phase modulation of a CW optical signal using two sequential EOMs. Analogous and spectrally identical to BPSK_PM_NRZ. Parameters are

- **TypeEOMElecData** with appropriate parameters (Recommendation: EOM_BINARY_DATA_RCS)
- **TypeEOMElecFilter** with appropriate parameters (Recommendation: EOM_NO_ELEC_FILTER)

Up to a constant factor, \( u \) is given by

\[
u(t) = \exp[i\pi(1 - V_1(t))] \exp\left[i\frac{\pi}{2}(1 - V_2(t))\right], \tag{12}\]

where \( V_1 \) and \( V_2 \) are two electrical signals of the type described above. So,

\[
u = \begin{cases} 
1 & \text{if } (V_1, V_2) = (1,1), \\
i & \text{if } (V_1, V_2) = (1,0), \\
-1 & \text{if } (V_1, V_2) = (0,1), \\
-i & \text{if } (V_1, V_2) = (0,0). 
\end{cases} \tag{13}\]

Transitions between data symbols lie on the unit circle.

**QPSK_DPMZ_NRZ**

Direct quadrature modulation of a CW optical signal using a dual-parallel MZ EOM. Analogous and spectrally identical to BPSK_MZ_NRZ Parameters are

- **TypeEOMElecData** with appropriate parameters (Recommendation: EOM_BINARY_DATA_RCOS)
- **TypeEOMElecFilter** with appropriate parameters (Recommendation: EOM_NO_ELEC_FILTER)
- \( \varepsilon = \text{MZM\_SubOptimalModulationDepthFactor} \) in range \( 0 \leq \varepsilon < 0.25 \). (\( \varepsilon = 0 \) is optimal case.)

Up to a constant factor, \( u \) is given by

\[
u(t) = \frac{1}{\sqrt{2}}(u_1(t) + iu_2(t)), \tag{14}\]

where

\[
u_k(t) = \sqrt{P_0} \cos \{\pi [1 - \varepsilon + (2\varepsilon - 1) V_k(t)]\}, \tag{15}\]

where \( V_k \) for \( k = 1,2 \) are independently generated electrical data streams. So, in case \( \varepsilon = 0 \),

\[
u = \begin{cases} 
\frac{1}{\sqrt{2}}(1 + i) & \text{if } (V_1, V_2) = (1,1), \\
\frac{1}{\sqrt{2}}(1 - i) & \text{if } (V_1, V_2) = (1,0), \\
\frac{1}{\sqrt{2}}(-1 + i) & \text{if } (V_1, V_2) = (0,1), \\
\frac{1}{\sqrt{2}}(-1 - i) & \text{if } (V_1, V_2) = (0,0). 
\end{cases} \tag{16}\]

In that case, transitions between data symbols go along the sides and diagonals of a square.
(B/Q)PSK *** RZ##

(B/Q)PSK *** NRZ followed by RZ## MZ in which \( t_{\text{NRZ}} \) is replaced by the (B/Q)PSK *** NRZ optical signal.

Summary for Group II

All formats use electrical data. If we assume this is of \( \text{TypeEOMElecData} = \text{EOM\_BINARY\_DATA\_RCOS} \) with \( \text{TypeEOMElecFilter} = \text{EOM\_NO\_ELEC\_FILTER} \) then we have parameters as follows for each of the 5 types of NRZ:

- NRZ_RCOS_MZ: EOM_RiseTime (Elec) or RiseTime (Opt); Extinction Ratio (Opt); Chirp (Opt)
- BPSK_PM_NRZ: EOM_RiseTime (Elec)
- QPSK_SEQ_PM_NRZ: EOM_RiseTime (Elec)
- BPSK_MZ_NRZ: EOM_RiseTime (Elec); \( \varepsilon = \text{MZM\_SubOptimalModulationDepthFactor} \)
- QPSK_DPMZ_NRZ: EOM_RiseTime (Elec); \( \varepsilon = \text{MZM\_SubOptimalModulationDepthFactor} \)

For each of these 5 formats there are NRZ, RZ33, RZ50 and RZ67 versions. No additional parameters are required for these.